CORDIAL LABELING FOR DIFFERENT TYPES OF SHELL GRAPH

S. Meena
Head of the department,
Department of mathematics,
Government Arts College,
C. Mutlur,
Chidambaram.
meenasaravanan14@gmail.com

M. Renugha
Assistant professor
Department of mathematics,
Krishnasamy college of Science,
Arts & Management for Women.
Cuddalore.
renughasrd@gmail.com

M. Sivasakthi
Assistant professor
Department of mathematics,
Krishnasamy college of Science,
Arts & Management for Women.
Cuddalore.
sivasakthi2203@gmail.com

Abstract— The aim of this paper is to introduce the cordial labeling for different types of shell graphs like path joining of shell graph, star of shell graph, multiple of shell graph, cycle of shell graph and to provide some results on it.

Keywords— Shell graph C (n, n-3), star of shell graph Sn , multiple shell graph, cycle of shell graph.

I. Introduction

We begin with simple, finite and undirected graph G= (V, E) with p vertices and q edges. For standard terminology and notation related to graph theory we refer to J.A. Gallian [2]. We will provide brief summary of definition and other information which are necessary for the present investigations.

II. Preliminaries

In the following we provide the essential definitions and results necessary for the development of our theory.

Definition 2.1[2]. If the vertices are assigned values subject to certain conditions then it is known as graph labeling.

Definition 2.2[2]. A mapping f: V (G) → {0, 1} is called binary vertex labeling of G and f(v) is called the label of the vertex V of G under f.

Definition 2.3[2]. Let f be a function from the vertices of G to {0,1} and for each edge xy assign the label f(x)-f(y). A cordial labeling of G if the numbers of vertices labeled 0 and the numbers of vertices labeled 1 differs by at most 1 and the numbers of edges labeled 0 and the number of edges labeled 1 differs by at most 1.

Definition 2.4[3]. A graph obtained by replacing each vertex of star graph $K_{1,n}$ by a graph G is called star graph of G. We denote it as G* is the graph which replaces central vertex of the graph $K_{1,n}$.

Definition 2.5. In a cycle C_n each vertex is replaced by a graph G is said to be cycle of graph.

Definition 2.6. A shell graph is defined as a cycle C_n with (n-3) chords sharing a common end point called the apex. Shell graph are denoted as $C_{(n,n-3)}$. A shell S_n is also called fan f_{n-1}.

Definition 2.7. A multiple shell is defined to be a collection of edge disjoint shells that have their apex is common.

ISSN 2229-5518

III. MAIN RESULTS

Theorem 3.1. Shell graphs are cordial.

Proof. Let G be a shell and let $v_1, v_2, \ldots v_n$ be successive vertices of G.
To define labeling function f:V(G)→{0,1}.
We consider the following cases

Case (i)

n ≡ 0(mod 4)

In this case we define labeling f as

f (u_i) = 0 ,if i ≡ 1,3 (mod 4)

= 1 , if i ≡ 0,2 (mod 4) , 1≤ i ≤ n

Case (ii)

n ≡ 1,2,3 (mod 4)

In this case we define the labeling f as

f (u_i) = 0 , if i ≡ 0,1 (mod 4)

= 1 , if i ≡ 2,3 (mod 4) , 1≤ i ≤ n

The labeling pattern defined above covers all possible arrangement of vertices.
In each case , the graph G under consideration satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.
That is, G admits a cordial labeling.

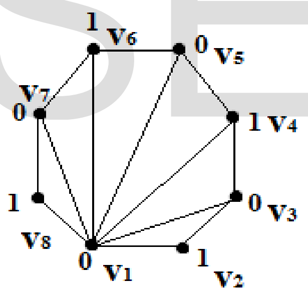

Fig1. Cordial shell graph

Theorem 3.2. The graph obtained by joining two copies of shell graph by a path of arbitrary length is cordial.

Proof. Let G be a graph obtained by joining two copies of shell graph by a path of length. Let $u_1, u_2, \ldots u_n$ be the successive vertices of 1st copy of shell graph and let $v_1, v_2, \ldots v_n$ be the successive vertices of 2nd copy of shell graph. Let $w_1, w_2, \ldots w_k$ be the successive vertices of path P_k with w_1 = u_1 and w_k = v_1.
To define the labeling function f : V(G) → {0,1}

f (u_i) = 0 , if i is odd

= 1 , if i is even.

f (v_i) = 0 , if i is odd

= 1 , if i is even.

f (w_k) = 0 , if k ≡ 1,3 (mod 4)

= 1 , if k ≡ 0,2 (mod 4).

The graph G under consideration satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. i.e., G admits a cordial labeling.

Example 3.3.

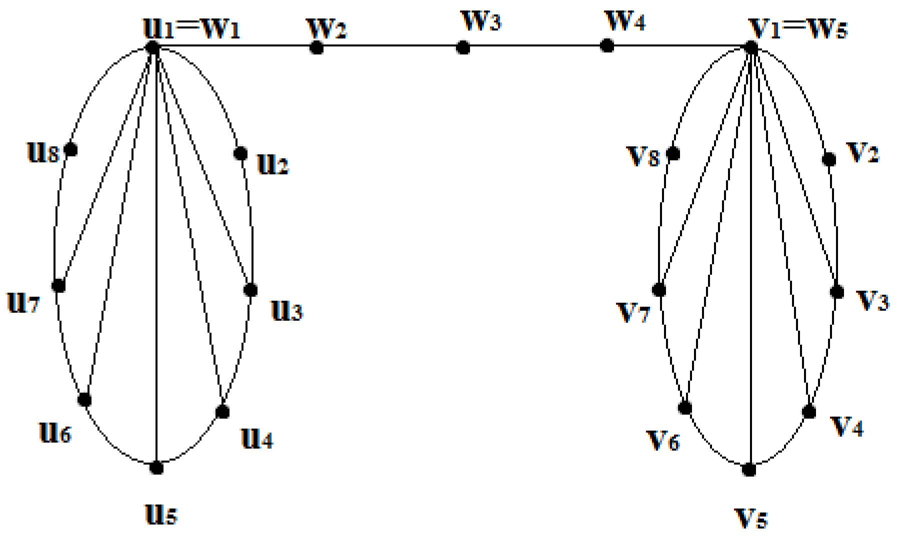

Fig2. Path joining of cordial shell graph

Theorem 3.4. Star of shell graph S_n* is cordial for all n.

Proof. Let $u_1, u_2, \ldots u_n$ be a successive vertices of central shell of S_n* and $v_{i1}, v_{i2}, \ldots v_{in}$ be successive vertices of other shells S_n(i) (except central shell), i = 1,2,…n. Let e_i be the edge such that $e_i = v_{i1}u_i$. Moreover let us denote the vertex of shell S_n(i) which is adjacent to vertex u_i labeled by 0 as v_{ij}(0) and similarly denote the vertex of shell S_n(i) which is adjacent to u_i labeled by 1 as v_{ij}(1).

To define the required labeling f : V(G) → {0,1}
We consider the following cases.

Case(i)

n ≡ 0 (mod 4)

In this case we define the labeling f as

f (u_i) = 0 , if i ≡ 0,1 (mod 4)
= 1 , if i ≡ 2,3 (mod 4) , 1≤ i ≤ n

f ($v_{ij}(0)$) = 0 , if j ≡ 0,3 (mod 4)
= 1 , if j ≡ 1,2 (mod 4) , 1≤ i ≤ n, 1≤ j ≤ n.

f ($v_{ij}(1)$) = 0 , if j ≡ 2,3 (mod 4)
= 1 , if j ≡ 0,1 (mod 4) , 1≤ i ≤ n, 1≤ j ≤ n.

Case (ii)

n ≡ 1 (mod 4)

In this case we define the labeling f as

f (u_i) = 0 , if i ≡ 0,1 (mod 4)
= 1 , if i ≡ 2,3 (mod 4) , 1≤ i ≤ n

f ($v_{ij}(0)$) = 0 , if j ≡ 0,3 (mod 4)

= 1 , if j ≡ 1,2 (mod 4) , 1≤ i ≤ n, 1≤ j ≤ n.
f ($v_{ij}(1)$) = 0 , if j ≡ 1,2 (mod 4)
= 1 , if j ≡ 0,3 (mod 4) , 1≤ i ≤ n, 1≤ j ≤ n.

Case (iii)
n ≡ 2 (mod 4)
In this case we define the labeling f as
f (u_i) = 0 , if i ≡ 0,2 (mod 4)
= 1 , if i ≡ 1,3 (mod 4) , 1≤ i ≤ n
f ($v_{ij}(0)$) = 0 , if j ≡ 0,1 (mod 4)
= 1 , if j ≡ 2,3 (mod 4) , 1≤ i ≤ n, 1≤ j ≤ n.
f ($v_{in}(1)$) = 1, f ($v_{in-1}(1)$) = 0 and
f ($v_{ij}(1)$) = 0 , if j ≡ 0,3 (mod 4)
= 1 , if j ≡ 1,2 (mod 4) , 1≤ i ≤ n, 1≤ j ≤ n–2.
Case (iv)
n ≡ 3 (mod 4)
In this case we define the labeling f as
f (u_i) = 0 , if i ≡ 0,1 (mod 4)
= 1 , if i ≡ 2,3 (mod 4) , 1≤ i ≤ n
f ($v_{ij}(0)$) = 0 , if j ≡ 0,1 (mod 4)
= 1 , if j ≡ 2,3 (mod 4) , 1≤ i ≤ n, 1≤ j ≤ n.
f ($v_{ij}(1)$) = 0 , if j ≡ 2,3 (mod 4)

= 1 , if j ≡ 0,1 (mod 4) , 1≤ i ≤ n, 1≤ j ≤ n.
The graph G under consideration satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.
i.e., G admits a cordial labelling.

Example 3.5. For better understanding of above above defined as labeling pattern , let us consider S_6*. The cordial labeling for S_6* is shown in the following figure.

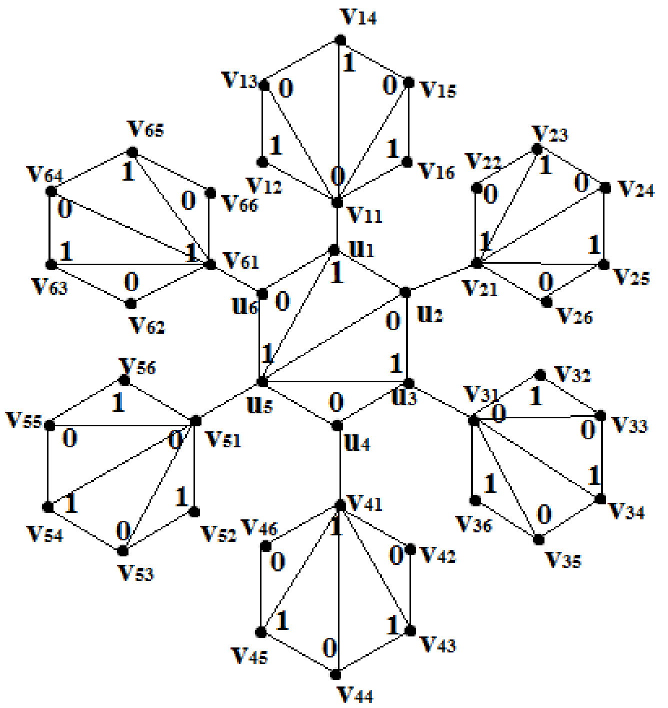

Fig3. Star of shell graph

Here $v_f(0) = 21$ and $v_f(1) = 21$. Therefore, $|v_f(0) - v_f(1)| \leq 1$.
S_6* satisfies the condition.
Therefore S_6* admits a cordial labeling.

Theorem 3.6. Multiple shell graph is cordial.
Proof. Let G be a multiple shell graph. Let the common (apex) vertex of multiple shell graph is v_0 and let $u_{i1}, u_{i2}, \ldots u_{in}$ are the successive vertices of shells. To define the required labeling f:V(G)→{0,1}. Let $f(v_0) = 1$

$f(u_{ij}) = 0$, if $j \equiv 1,3 \pmod 4$
$= 1$, if $j \equiv 0,2 \pmod 4$

The labeling pattern defined above covers all possible arrangement of vertices.

The graph G under consideration satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. That is, G admits a cordial labeling

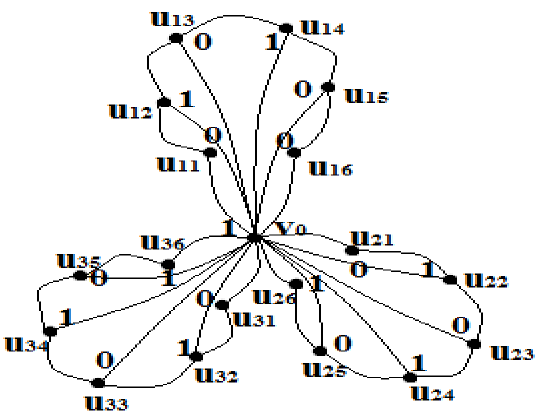

Fig4. Multiple shell graph

Theorem 3.7. The cycle of shell graph is cordial.

Proof. Let G be a cycle of shell graph and let $u_{i1}, u_{i2}, \ldots u_{in}$ are the successive vertices of $1^{st}, 2^{nd}, \ldots n^{th}$ copy of the shells. To define the labeling function f:V(G)→{0,1}. We consider the following cases.

Case (i)

If i is odd then

$f(u_{ij}) = 0$, if j is odd

$= 1$, if j is even, $1 \leq i \leq n$.

Case (ii)

If i is even then

$f(u_{ij}) = 1$, if j is odd

$= 0$, if j is even, $1 \leq i \leq n$.

The labeling pattern defined above covers all possible arrangement of vertices.

In each case , the graph G under consideration satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

i.e., G admits a cordial labelling

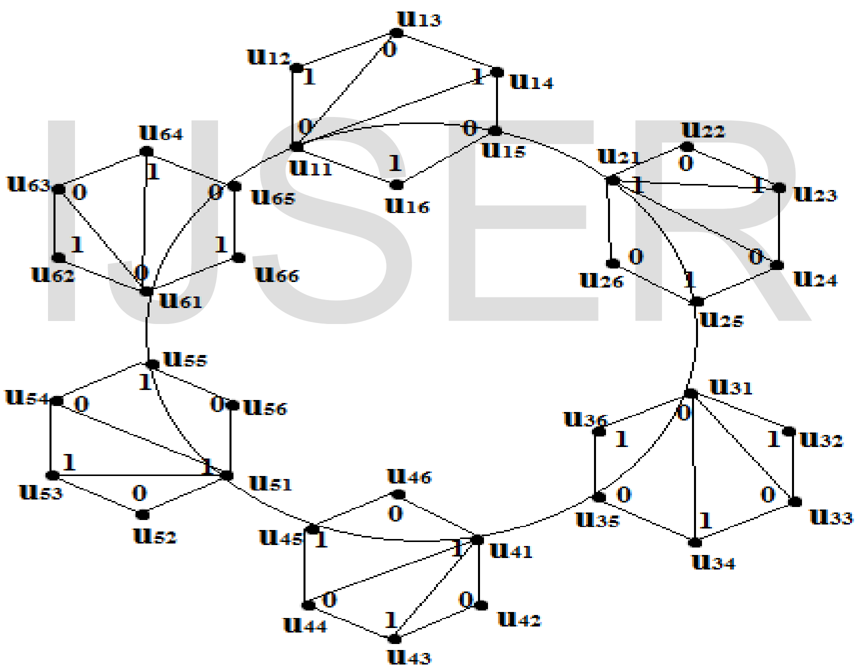

Fig5. Cycle of shell graph is cordial

IV. Conclusions

As all the shell graphs are cordial graphs it is very interesting and challenging as well to investigate cordial labeling for star shell graph, Multiple shell graph, cycle of shell graph, which admit Cordial labeling. Here we have contributed some new result is by investigating cordial labeling for cycle of shell graph and multiple shell graph.

Acknowledgment

We are blessed for having it guided by the almighty and also thankful to all those who motivated us. We extend our sincere thanks to Mr. S. Sivaramakrishnan, Assistant Professor of Mathematics and Research Scholar, Krishnasamy college of Engineering & Technology, Cuddalore, for being a good support to finish this work.

References

ISSN 2229-5518
[1] F. Harary, Graph Theory, *Addison Wesley*, Reading, MA, 1969.
[2] J. A. Gallian, *A dynamic survey of graph labeling*, The Electronics Journal of Combinatorics, DS6, 16, 2009.
[3] S.K. Vaidya, N.A. Dani, K.K.Kanani and P.L. Vihol, *Cordial and 3-Equitable Labeling for some shell related graphs*, Journal of scientific research , J.Sci.Res.1(3), 438-449,2009.
[4] I. Cahit, Corrdial graphs : *A weaker version of graceful and harmonious graphs*, Ars Combin. 23 (1987), pp. 201−207.
[5] I. Cahit, *On cordial and 3−equitable labellings of graphs*, Utilitas. Math. 37 (1990), pp. 189−198.
[6] Y. S. Ho, S. M. Lee and S. C. Shee, *Cordial labeling of unicyclic graphs and Generalized Petersen graphs*, Congress Number. 68, (1989), pp. 109−122.
[7] M. Z. Youssef, *On skolem graceful and cordial graphs*, Ars Combinatorics, 78 (2006), pp. 167−177.
[8] S. K. Vaidya, Sweta Srivastav, V. J. Kaneria and G. V. Ghodasara, *Cordial and 3−equitable labeling of cycle with twin−chords*, Proceeding of The International Conference on Emerging Technologies and Applications in Engineering, Technology and Sciences 13−14 Jan.(2008), Vol−I, pp. 905−907.
[9] S. K. Vaidya, Sweta Srivastav, V. J. Kaneria and G. V. Ghodasara, *Some new cordial graphs*, International J. of Scientific Computing, 2(1), (2008), pp. 81−92.
[10] S. K. Vaidya, Sweta Srivastav, V. J. Kaneria and G. V. Ghodasara, *Cordial labeling for cycle with one chord and its related graphs*, Indian J. of Math. and Math. Sc. Vol−4(2), (2008), pp. 145−156.
[11] S. K. Vaidya, Sweta Srivastav, V. J. Kaneria and G. V. Ghodasara, *Cordial labeling for two cycle related graphs*, The Math. Student J. of Indian Math. Soc. 76 (2007), pp. 237−246.
[12] S. K. Vaidya, Sweta Srivastav, V. J. Kaneria and G. V. Ghodasara, *Cordial and 3−equitable labeling of star of a cycle*, Mathematics Today 24 (2008), pp. 54−64.